Analysis and design of steel structures for buildings according to Eurocode 0, 1 and 3

Steel Design 1

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Structural basics

Colophon

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The text is based on the (English) EN version of the Eurocodes using default and/or recommended values. Where a country can make a national choice – or when non-contradictory complementary information may be used – this is indicated by the following symbol: NA. Separate annexes contain the national choices for Belgium, Germany, Luxembourg, The Netherlands and Switzerland. These annexes can be downloaded free of charge from the websites of the (national) organisations as well as any errata, corrections and additions to this textbook.

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Content

Structural basics

1

Structural safety

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1 Structural safety

The safety of structures – of buildings in which we live and work amongst others – is a fundamental need of humanity. In many countries, the government sees it as its responsibility to guarantee structural safety. Structural safety is mostly addressed by legal regulations which designate the building standards and codes to be used, in particular the Eurocodes. In this way minimum requirements for the safety of structures are assured. Buildings and bridges (or parts of these) can collapse when their structural elements do not satisfy these minimum requirements, leading to significant damage or even casualties (fig. 1.1 and 1.2). Many publications, amongst others [1], discuss structural safety and reliability.
The general principles of structural safety are presented in the basic Eurocode EN 1990. This code describes the principles of structural design and analysis and provides guidelines for interdependent aspects of the structural reliability.

This chapter first discusses the theory and background of EN 1990 regarding:
– probability of failure (safety);
– reliability principles (taking uncertainties into account);
– design value of resistance (strength of a structure);
– design value of actions (action types and combinations of actions);
– reliability (consequence classes and reliability index).

Finally, the content and structure of EN 1990 is discussed briefly, following the order of the chapters in the code.

1.1 Collapsed parking deck at a hotel in Tiel (The Netherlands, 2002) due to, amongst other things, insufficient stability of the edge beam.

1.2 Collapsed Saint Anthony Falls Bridge in Minneapolis (USA, 2007) due to incorrectly designed joints (gusset plates) in the truss.

1.1 Probability of failure

When designing a structure, the structural engineer needs to show that the effect of actions E on the structure is lower than the resistance R of the structure during its design working life. The term 'actions' is broad, covering not only loads but also, for example, imposed deformations, and expansion due to changing temperature and creep.
The effect of actions on a structure depends on the following basic variables:
- actions and environmental influences;
- material and product properties;
- geometrical properties of the structure and its elements.

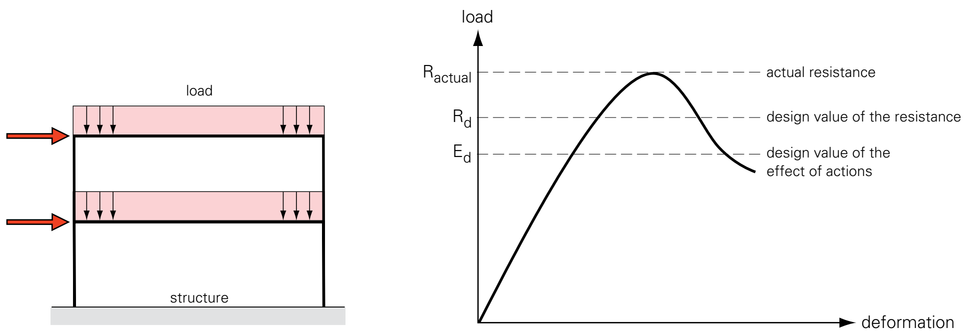
Using applied mechanics the effect of actions can be described in terms of internal forces – such as bending moments, shear forces and normal forces – or, for example, stresses, strains or deflections. After determining the geometrical properties of the structure – including the cross-section dimensions – the cross-section properties can be determined from books of tables, and the magnitude of the actions is determined using the different parts of the Eurocode on actions, EN 1991, see *Structural basics 2* (Actions and deformations). The material properties for steel structures follow from EN 1993-1-1. Thus, using this approach, all basic variables get one specific value.
The assessment procedure for structures is in this way similar as a deterministic approach. The structural engineer should appreciate that many basic variables in reality do not have the exact same values as those applied in the analysis. This is due to the fact that all basic variables are, statistically speaking, so-called stochastic variables: actions vary in time, dimensions vary between tolerance limits and material properties show certain variability. The structural engineer should therefore show that the probability of failure of the structure is sufficiently small. By following the Eurocode approach the engineer will implicitly ensure that the probability of failure is sufficiently small.
The probability of failure of a structure is denoted as P_f. The probability of survival P_s is the probability that the structure does not fail and is complementary to the probability of failure. According to the theory of probability, the sum of the probability of failure and the probability of survival is equal to one. The probability of survival is referred to as the reliability. The reliability of the structure is then:

$$P_s = 1 - P_f \tag{1.1}$$

The result of a reliability analysis is the probability that the structure survives, which is known as its reliability. This probability of survival is generally almost equal to 1, for example in the order of P_s = 0,999999, where the number 1 is equal to 100%. This can also be written as:

$$P_s = 0{,}999999 = 1 - 0{,}000001 = 1 - 10^{-6} \tag{1.2}$$

Equation (1.2) shows that the probability of failure is $P_f = 10^{-6}$. In the interests of clarity the result of a reliability analysis, although defining reliability, is usually presented as a probability of failure.



1.4 Assessment method for the ultimate limit state.

This requirement is often presented as a unity check in the Eurocodes:

$$\frac{E_d}{R_d} \leq 1 \tag{1.5}$$

Where:

E_d design value of the effect of actions ($E_d = \gamma_F E_k$);
R_d design value of the corresponding resistance ($R_d = R_k/\gamma_M$);
E_k characteristic value of the effect of actions;
R_k characteristic value of the resistance;
γ_F partial factor for actions; $\gamma_F \geq 1$;
γ_M partial factor for resistance; $\gamma_M \geq 1$.

In this way it is possible to assess the reliability of the structure (fig. 1.4).

1.3 Design value of resistance

The design value of resistance of the structure follows from the resistance function of the design model. This model is based on a combination of theoretical considerations and the observed behaviour of structures during tests. The resistance function R of a member loaded in tension is, for example:

$$R = Af_y \tag{1.6}$$

Where:

A area of the cross-section;
f_y yield stress.

β	1,28	2,32	3,09	3,72	4,27	4,75	5,20	5,61
P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}

1.5 Relationship between the reliability index β and the probability of failure P_f.

The suitability of a chosen design model can be assessed by comparing results of the resistance function with test results, for example of members loaded in tension. The design model is adjusted until sufficient agreement is reached between the theoretical results and test results. When this is the case, the resistance function R is adjusted and the characteristic strength R_k can be determined, expressed in terms of the nominal value for the dimensions and the design value of the material properties. Finally, the partial factor for resistance γ_M is applied and the design value of the resistance R_d follows from:

$$R_d = \frac{R_k}{\gamma_M} \qquad (1.7)$$

The value of γ_M depends on the reliability index β, which is a measure for the probability of failure of the structure. The relationship between the value of the reliability index β and the probability of failure P_f is shown in table 1.5. EN 1990 assumes $\beta = 3{,}8$ for the ultimate limit state, and for a reference period of fifty years.

When the procedure described above for determining the design value of resistance is applied strictly, it will lead to a different value of γ_M for each resistance function. This is clearly inconvenient

NA in practice. Therefore, EN 1993-1-1, cl. 6.1 provides a limited number of recommended values for buildings, depending on the nature of failure:

$\gamma_{M0} = 1{,}00$ for cross-sections where yielding governs and therefore the yield stress is of importance in the resistance function;

$\gamma_{M1} = 1{,}00$ for stability of members;

$\gamma_{M2} = 1{,}25$ for cross-sections loaded in tension up to fracture, where fracture governs and therefore the tensile strength is of importance in the resistance function.

1.4 Design value of actions

It is important to consider not only the resistance but also the actions in order to assess the reliability of a structure. However the actions cannot be described by only a characteristic value in combination with a certain probability of exceedance. Therefore, representative values are used for the actions, see also sections 1.4.3, 1.6.4 and 1.6.6.

Not only are there several types of actions, but the action which should be taken into account depends on the location of the structural member. For a beam which supports a roof for example, snow load should be included in the combination of actions. However, for a beam which supports a floor snow load is irrelevant, and the imposed floor load should be taken into account in the combination of actions.

2.4.2 Snow

The load due to snow is specified in EN 1991-1-3. For persistent and transient design situations, the snow load which should be taken into account can be determined by:

$$s = \mu_i C_e C_t s_k \qquad (2.9)$$

Where:

μ_i snow load shape coefficient depending on the shape of the roof;
C_e exposure coefficient;
C_t thermal coefficient;
s_k characteristic value for snow load on the ground

The exposure coefficient C_e should be used for determining the snow load on the roof. The choice for C_e should consider the future development around the site. C_e should be taken as 1,0 unless otherwise specified for different topographies. The National Annex may give the values of C_e for different topographies. The recommended values are given in table 2.14.

a NA 2.14 Recommended exposure coefficients C_e.

topography	C_e
windswept[a]	0,8
normal[b]	1,0
sheltered[c]	1,2

a. Windswept: flat unobstructed areas exposed on all sides without or with little shelter.
b. Normal: areas without significant removal of snow by wind.
c. Sheltered: areas considerably lower than surrounding terrain, high trees or higher buildings.

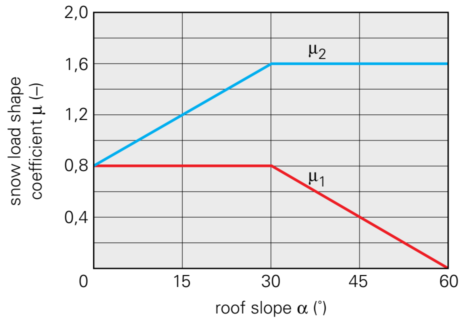
The thermal coefficient C_t may be used to account for the reduction of snow loads on roofs with high thermal transmittance (> 1 W/m^{2}K), in particular for some glass covered roofs, because of melting caused by heat loss. For all other cases: C_t = 1,0.

a NA The characteristic value for snow load on the ground s_k is specified in the National Annexes of the different countries.
Accompanying values of the snow load are set to $\psi_i \cdot s_k$, with ψ_0 for the combination value, ψ_1 for the frequent value and ψ_2 for the quasi-permanent value. Recommended values for these ψ coefficients are given in table 2.15. This results in the representative values of the snow load for simultaneous occurrence with other variable actions.
b NA Exceptional snow loads on the ground can be determined using EN 1991-1-3, cl. 4.3.

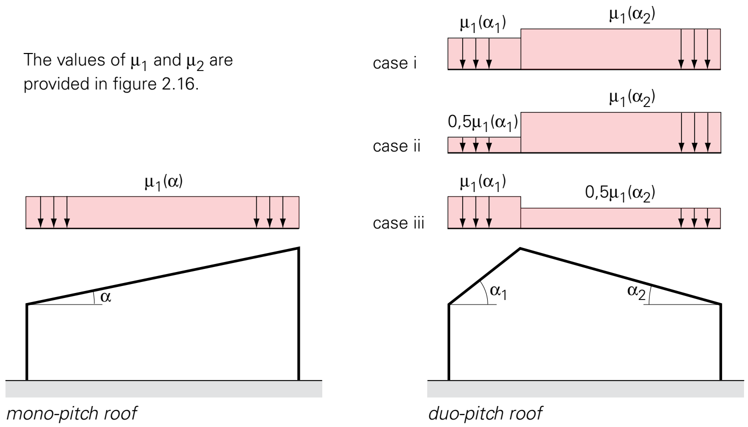
a NA 2.15 Recommended ψ coefficients for accompanying snow loads.

regions	ψ_0	ψ_1	ψ_2
Finland, Iceland, Norway, Sweden	0,70	0,50	0,20
other CEN member states for sites located at altitude H > 1000 m above sea level	0,70	0,50	0,20
other CEN member states for sites located at altitude H 1000 m above sea level	0,50	0,20	0,00

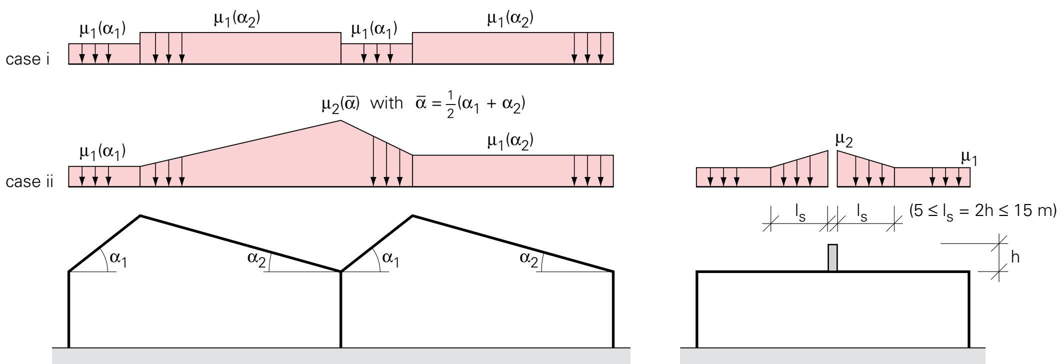
EN 1991-1-3, cl. 5.3, provides values for the snow load shape coefficient μ for mono-pitch roofs, duo-pitch roofs and multi-span roofs (fig. 2.16). For mono-pitch and duo-pitch roofs, figure 2.17 provides the snow load arrangements which should be considered. For the duo-pitch roof, case i is the undrifted snow load arrangement;
c NA cases ii and iii are the drifted snow load arrangements. Figure 2.18 shows the load arrangements for multi-span roofs and for obstacles. EN 1991-1-3 also provides snow load shape coefficients and load arrangements for cylindrical roofs and for roofs close to taller building structures.



2.16 Snow load shape coefficient for mono-pitch roofs, duo-pitch roofs and multi-span roofs.



2.17 Load arrangements for snow on pitched roofs.



a. multiple-span roofs (the values of μ_1 and μ_2 are provided in figure 2.16)

b. obstacels: μ_1 = 0,8 and $\mu_2 = \gamma h/s_k$ with $0{,}8 \leq \mu_2 \leq 2{,}0$ and γ is the weight density of snow (γ = 2 kN/m^3)

2.18 Load arrangements for snow on multi-span roofs and for obstacles.

2.4.3 Wind

Wind action is considered in EN 1991-1-4. This standard is not easy to understand, so only the most important aspects of EN 1991-1-4 are discussed below.

Wind is moving air with a density of about 1,25 kg/m^3, and both the velocity and the direction vary in time. Analyses of the wind action on structures assume a wind velocity which on average is exceeded once per fifty years: this is referred to as the fundamental value of the basic wind velocity $v_{b,0}$. This is the characteristic 10 minutes mean wind velocity at 10 m height above the ground in an open area. The value of $v_{b,0}$ differs by region. Usually, multiple wind velocities are used per country in

NA Europe as specified in the different National Annexes.

Example 3.1

- *Given*. A T-section formed from half a HEB 300 section (fig. 3.19).
- *Question*. Determine shape factor α_y for the y-axis.
- *Answer*. If it is assumed that the plastic neutral axis is located in the flange of the section, the height of the plastic zone under the plastic neutral axis $e_{z.pl}$ is (fig. 3.19):

$$e_{z,pl} b = \frac{1}{2} A$$

Rewritten and calculated:

$$e_{z,pl} = \frac{A}{2b} = \frac{7454}{2 \cdot 300} = 12,4 \text{ mm}$$

The distance z_1 of the plastic neutral axis to the centre of gravity Z_1 of the part of the section above the plastic neutral axis – neglecting the root radii (fillets) – is:

$$z_1 = \frac{0,5b\left(t_f - e_{z,pl}\right)^2 + \left(h - t_f\right)t_w\left(t_f - e_{z,pl} + 0,5\left(h - t_f\right)\right)}{0,5A}$$

$$= \frac{0,5 \cdot 300 \cdot \left(19 - 12,4\right)^2 + \left(150 - 19\right) \cdot 11 \cdot \left(19 - 12,4 + 0,5 \cdot \left(150 - 19\right)\right)}{0,5 \cdot 7455} = 29,6 \text{ mm}$$

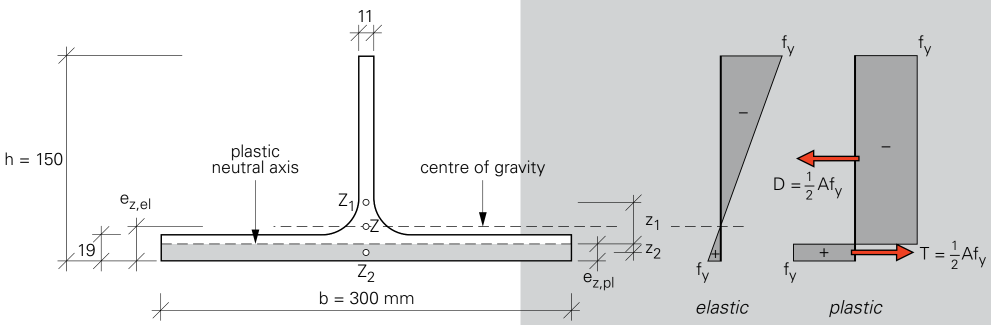
The distance z_2 of the plastic neutral axis to the centre of gravity Z_2 of the part of the section below the plastic neutral axis is:

$$z_2 = 0,5 e_{z,pl} = 0,5 \cdot 12,4 = 6,2 \text{ mm}$$

The plastic section modulus $W_{pl,y}$ is the sum of the first moments of area of both parts of the section with respect to the plastic neutral axis:

$$W_{pl,y} = S_1 + S_2 = \frac{1}{2} A\left(z_1 + z_2\right) = \frac{1}{2} \cdot 7455 \cdot \left(29,6 + 6,2\right) = 133 \cdot 10^3 \text{ mm}^3$$

3.19 T-section formed from half a HEB 300 section.



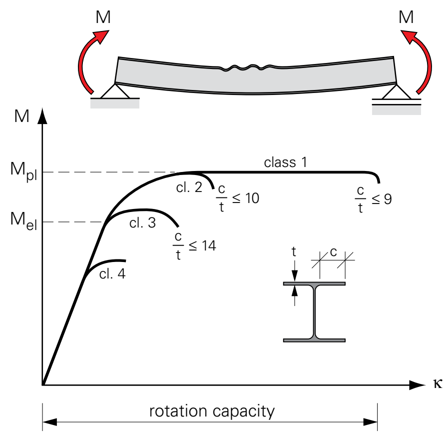
If the root radii are taken into account, the plastic section modulus is $W_{pl,y} = 137 \cdot 10^3$ mm^3 and the elastic section modulus $W_{el,y} = 69,6 \cdot 10^3$ mm^3. Then the shape factor is:

$$\alpha_y = \frac{W_{pl,y}}{W_{el,y}} = \frac{137 \cdot 10^3}{69,6 \cdot 10^3} = 1,97$$

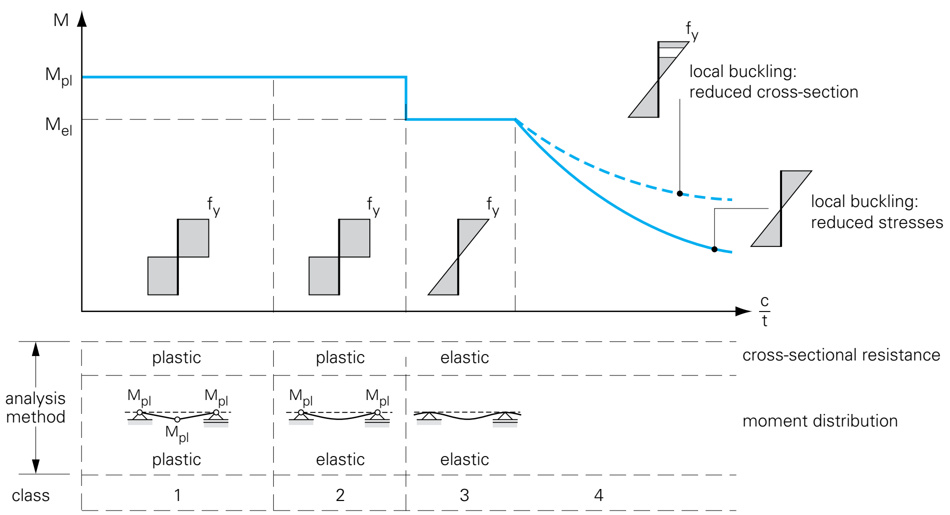
3.2.2 Classification

The properties of sections loaded in combined compression and bending depend to a large extent on the c/t ratios, where c is the width and t is the thickness, of the compressed plates forming the section. This can be seen from the schematic M/κ diagrams of an I-section loaded in bending, shown in figure 3.20. The shape of the M/κ diagram depends on the c/t ratio of the flange in compression, where c is measured from the root radius to the end of the flange. The load bearing capacity of the flange and the curvature at failure – thus also the rotation capacity – increase when the flange is stockier, that is has a greater relative thickness. This behaviour depends on to what extent the compression flange is susceptible to local buckling.

When choosing a global analysis method, the structural engineer must anticipate the cross-sectional deformation behaviour. For example, plastic theory may only be used when the sections have sufficient deformation capacity so that the assumed redistribution of moments can occur. This means that the rotation capacity – more generally the deformation capacity – has to be sufficiently large. Also, a distinction can be made between sections where local buckling occurs in the elastic range and sections where local buckling occurs in the plastic part of the behavioural response. For the former, a local buckling analysis is required; for the latter this is not necessary.
Comparing the deformation capacity of a cross-section with the deformation capacity required in a given situation is too complicated for everyday design. Therefore the Eurocode uses a classification system whereby cross-sections are subdivided into one of four classes depending on the plate dimensions and the yield stress (EN 1993-1-1, cl. 5.5). The global analysis methods, which are admissible for the different cross-section classes, are shown schematically in figure 3.21. The moment resistance, which can be achieved for pure bending, is also shown in this figure. The four cross-section classes are described below, each with a corresponding analysis method.



3.20 Influence of the c/t ratio of the flange of a rolled I-section in S235 on the M/κ diagram.



3.21 Relationship between cross-section class and analysis methods.

4.4.2 Joint properties

The discussion above was based on the assumption that the joint between the column and the beam of the frame is infinitely stiff ($S_j = \infty$) and that the strength of the joint does not govern ($M_{j,Rd} > M_{pl,Rd,cln}$). In such cases the response of the structure depends only on the properties of the beams and columns. In practice, these assumptions are never valid and more realistic properties of the joint should be used. A typical M/ϕ response of a joint may be considered to be bilinear. The following three different joints in the frame are considered below:

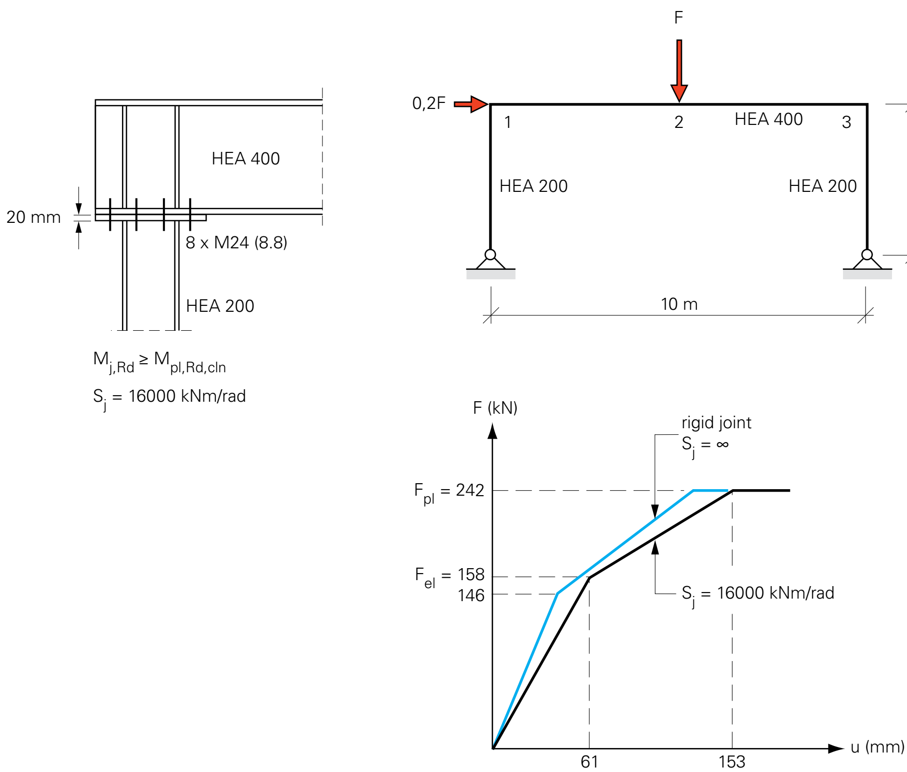
- extended header plate joint and pinned column base joint;
- flush header plate joint and pinned column base joint;
- flush header plate joint and semi-rigid column base joint.

Extended header plate joints and pinned column base joints

For the frame shown in figure 4.18, the joints between the columns (HEA 200) and the beam (HEA 400) adopt an extended header plate (fig. 4.23). The resistance of this type of joint is larger than the plastic moment resistance of the column ($M_{j,Rd} > M_{pl,Rd,cln}$), and the stiffness of the joint can be determined using software: S_j = 16000 kNm/rad.

For the analysis of the frame response, nodes 1 and 3 are represented by rotational springs with

4.23 Frame with beam-to-column joints with an extended header plate and pinned column base joints.



a known stiffness S_j. If the software used for the frame analysis cannot include springs, it is also possible to model a short dummy member corresponding with half the height of the beam, here with a length L_{bar} = 195 mm (fig. 4.24). This member is allocated properties – expressed in terms of its second moment of area I_{bar} – that ensure when subject to a moment it gives the same M/ϕ response as the equivalent spring.

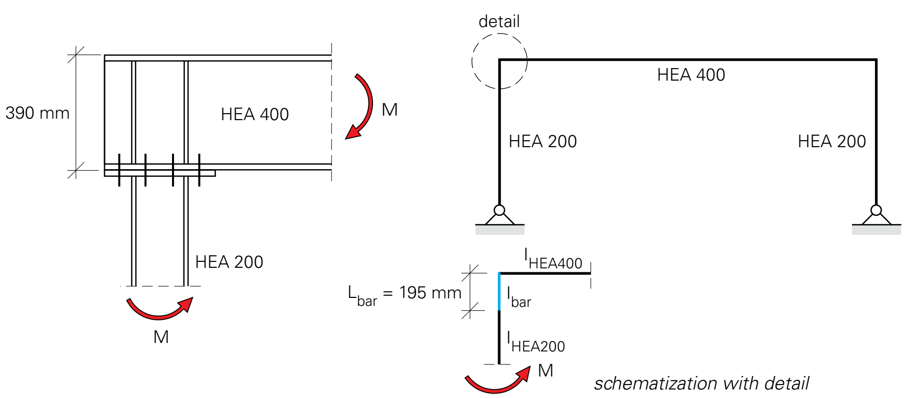
I_{bar} is determined as follows. The rotation ϕ of the member subject to a moment M is $\phi = ML_{bar}/EI_{bar}$ so that the fictional stiffness of the member is:

$$S_{bar} = \frac{M}{\phi} = \frac{EI_{bar}}{L_{bar}} \tag{4.21}$$

An equal stiffness of spring and member is obtained when $S_{bar} = S_j$. Therefore:

$$I_{bar} = \frac{S_j L_{bar}}{E} = \frac{16000 \cdot 10^6 \cdot 195}{2{,}1 \cdot 10^5} = 1486 \cdot 10^4 \ \text{mm}^4$$

The F/u diagram for the example frame with these joints is shown in figure 4.23. For reasons of comparison the F/u diagram of the structure with rigid joints is also shown. In both cases the failure load is the same, namely F_{pl} = 242 kN. This is logical because a first-order failure load only depends on strength and not on stiffness. The stiffness of the joints however does influence the elastic moment distribution, and therefore also the maximum load according to elastic theory. Figure 4.23 shows that lowering the stiffness of the joint leads, for the example frame, to an increase in F_{el}, namely from 146 kN to 158 kN.



4.24 Schematization of the beam-to-column joint from figure 4.23 by a fictive bar.

5.1 Linear elastic analysis (LA) and materially nonlinear analysis (MNA)

This section discusses method A (linear elastic analysis; LA) and method B (materially nonlinear analysis; MNA) for a two-bay unbraced frame. The importance of rotation capacity and the influence of the stiffness ratios in the structure, and the influence of the initial sway imperfection, are subsequently discussed.

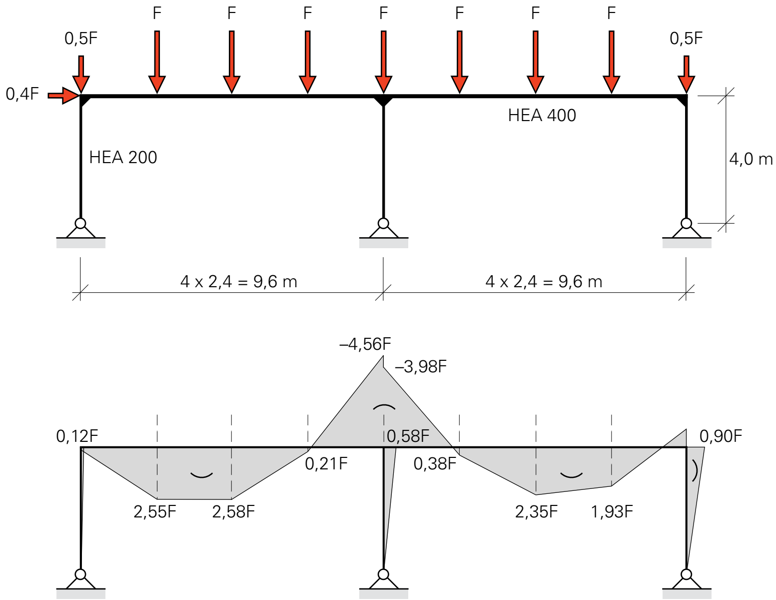
5.1.1 Unbraced two-bay frame

The application of first-order elastic and plastic analyses is illustrated for an unbraced two-bay frame (fig. 5.2), see also *Structural basics 4* (Analysis), section 4.3. It is assumed that the columns are rigidly connected to the beams ($S_j = \infty$) and that the joints have full-strength, meaning that the joint resistance is at least equal to the plastic moment resistance of the weaker of either the column or the beam section; here the column section is the weakest of the two, so $M_{j,Rd} \geq M_{pl,Rd,cln}$. The plastic moment resistances of the beam and the column, both of steel grade S235, are:

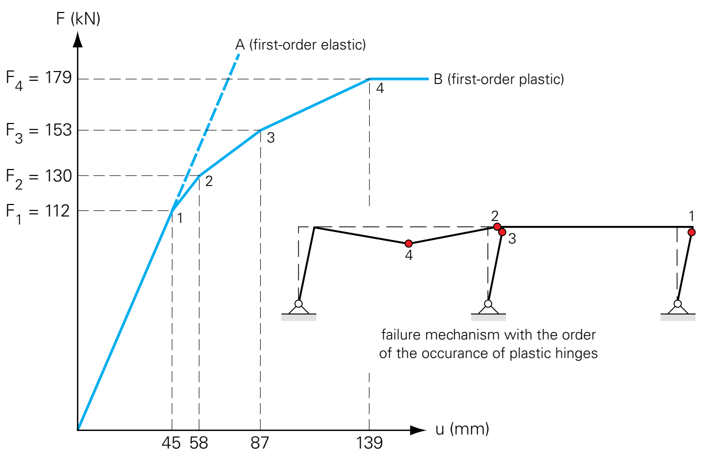
$$M_{pl,Rd,bm} = \frac{W_{pl} f_y}{\gamma_{M0}} = \frac{2562 \cdot 10^3 \cdot 235 \cdot 10^{-6}}{1,0} = 602 \text{ kNm}$$

$$M_{pl,Rd,cln} = \frac{W_{pl} f_y}{\gamma_{M0}} = \frac{430 \cdot 10^3 \cdot 235 \cdot 10^{-6}}{1,0} = 101 \text{ kNm}$$

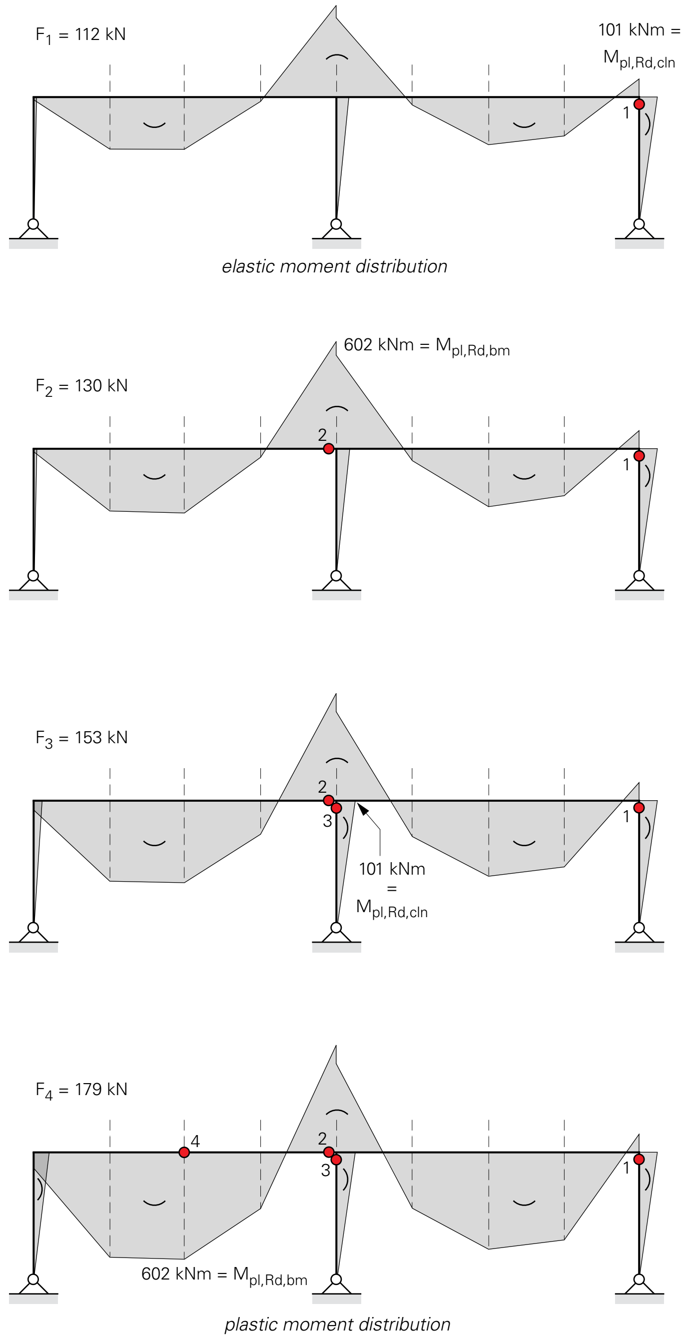
The moment diagram in figure 5.2 is determined from a linear elastic analysis. The first plastic hinge occurs when the elastic limit is reached in the right hand side column of the frame, if: $0{,}90F_1 = M_{pl,Rd,cln} = 101$ kNm, so $F_1 = 112$ kN. As the load is increased further, three other plastic hinges occur successively until a failure mechanism forms. The number of plastic hinges needed to form a plastic failure mechanism is equal to the degree n to which a structure is statically indeterminate plus one: n + 1. In this case: 3 + 1 = 4. Figure 5.3 shows how the four plastic hinges occur successively.
The bending moment diagrams can be determined using software which allows for an elastic-plastic option. They can however also be determined when using an elastic frame program. In that case, the elastic moment distribution should be determined for each increment of the load, and after the occurrence of a plastic hinge the static system should be modified by adding a hinge at that location. This approach is discussed in *Structural basics 4* (Analysis).
Figure 5.4 shows the relationship between the magnitude of the load F and the horizontal displacement u of the beams in the unbraced two-bay frame example. The dashed line A is obtained with a first-order elastic analysis (method A), and solid line B with a first-order plastic analysis (method B).



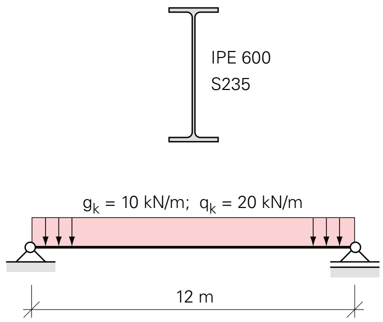
5.2 Unbraced two-bay frame with linear-elastic moment diagram.



5.4 Relationship between the load and the horizontal displacement of the frame from figure 5.2.



5.3 Moment distribution in the frame of figure 5.2 at the occurrence of the four successive plastic hinges (steel grade S235).



6.37 Roof girder loaded in bending.

Example 6.2 (bending)

- *Given*. A roof girder IPE 600 in steel grade S235 loaded by a uniformly distributed load g_k = 10 kN/m for self-weight and q_k = 20 kN/m for variable actions (fig. 6.37). Assume the partial factors for the loading are γ_g = 1,2 for the self-weight and γ_q = 1,5 for the variable actions.
- *Question*. Check the moment resistance of the beam.
- *Answer*. From example 6.1 it can be concluded that the section IPE 600 in steel grade S235 is class 1. This means that the check concerning the resistance may be performed considering plastic theory. The design value of the load is:

$$q_{Ed} = \gamma_g g_k + \gamma_q q_k = 1{,}2 \cdot 10 + 1{,}5 \cdot 20 = 42{,}0 \text{ kN/m}$$

The design value of the bending moment is:

$$M_{Ed} = \frac{1}{8} q_{Ed} L^2 = \frac{1}{8} \cdot 42{,}0 \cdot 12^2 = 756 \text{ kNm}$$

The plastic moment resistance may be used because the section is in class 1:

a NA

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl} f_y}{\gamma_{M0}} = \frac{3512 \cdot 10^3 \cdot 235}{1{,}00} = 825 \text{ kNm}$$

Unity check:

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{756}{825} = 0{,}92 \leq 1{,}0 \quad \text{(OK)}$$

Although the check concerning moment resistance is now complete, for comparison assume that the section would have been of class 3. Then, the elastic moment resistance should have been used:

b NA

$$M_{c,Rd} = M_{el,Rd} = \frac{W_{el} f_y}{\gamma_{M0}} = \frac{3069 \cdot 10^3 \cdot 235}{1{,}00} = 721 \text{ kNm}$$

The unity check would have been:

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{756}{721} = 1{,}05 > 1{,}0 \quad \text{(not OK)}$$

Then, in this case, a larger section would need to have been chosen.

Shear (elastic)

According to elastic theory, the shear stress τ resulting from a shear force can be determined as follows:

$$\tau = \frac{V_{Ed} S}{It} \qquad (6.15)$$

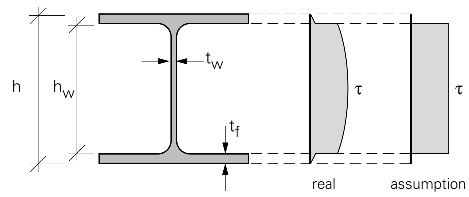
Where:

V_{Ed} design value of the shear force;

S first moment of area (static moment) about the centroidal axis of that portion of the cross-section between the point at which the shear is required and the boundary of the cross-section;

I second moment of area of the whole cross-section;

t thickness of the part of the section where the shear stress is required (for the web $t = t_w$).



6.38 Shear stress distribution for an I-section.

Figure 6.38 shows the distribution of shear stresses for an I-section loaded by a vertical shear force. The shear force is mainly resisted by the web of the cross-section. It is often assumed that the shear stresses are distributed uniformly over the web, in which case the following applies:

$$\tau = \frac{V_{Ed}}{A_w} \qquad (6.16)$$

In this equation A_w is the shear area of the web, measured between the flanges without the root radii (fig. 6.39a):

$$A_w = h_w t_w \qquad (6.17)$$

Where:

NA h_w height of the web measured between the flanges ($h_w = h - 2t_f$);

t_w thickness of the web.

The shear stress must not exceed the yield shear stress τ_y, which can be determined as:

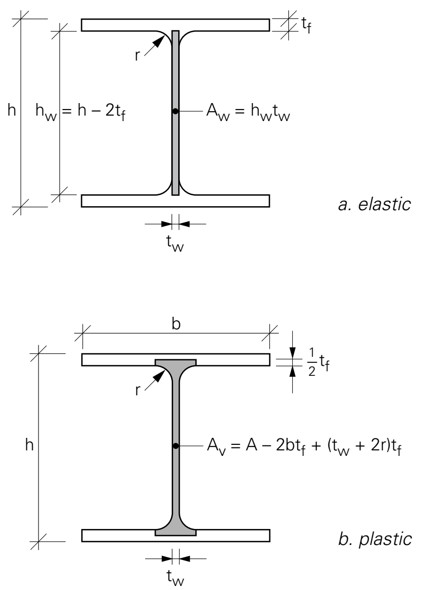
$$\tau_y = \frac{f_y}{\sqrt{3}} \qquad (6.18)$$

The check below needs to be satisfied:

$$\frac{V_{Ed}}{A_w} \leq \frac{f_y}{\sqrt{3}} \qquad (6.19)$$

Equation (6.19) can also be written as a unity-check:

$$\frac{V_{Ed}}{\frac{A_w f_y}{\sqrt{3}}} \leq 1{,}0 \qquad (6.20)$$



6.39 Shear area of an I-section.

Where:
V_{Ed} design shear force;
$V_{pl,T,Rd}$ reduced plastic design shear resistance allowing for the presence of torsional moment:

I- of H-section: $$V_{pl,T,Rd} = \sqrt{1 - \frac{\tau_{t,Ed}}{1,25\left(f_y/\sqrt{3}\right)/\gamma_{M0}}}\, V_{pl,Rd} \qquad (7.34)$$

channel section: $$V_{pl,T,Rd} = \left(\sqrt{1 - \frac{\tau_{t,Ed}}{1,25\left(f_y/\sqrt{3}\right)/\gamma_{M0}}} - \frac{\tau_{w,Ed}}{\left(f_y/\sqrt{3}\right)/\gamma_{M0}}\right) V_{pl,Rd} \qquad (7.35)$$

hollow section: $$V_{pl,T,Rd} = \left(1 - \frac{\tau_{t,Ed}}{\left(f_y/\sqrt{3}\right)/\gamma_{M0}}\right) V_{pl,Rd} \qquad (7.36)$$

7.4 Combined internal forces

The individual internal forces which can be present at a cross-section are discussed in section 7.3. In many cases, these internal forces can be present in combinations and the cross-section also needs to be checked for these. The following combinations of internal forces are discussed below, according to EN 1993-1-1, cl. 6.2.8 till 6.2.10:
– bending and shear;
– bending and axial force;
– bending, shear and axial force.

7.4.1 Bending and shear

NA EN 1993-1-1, cl. 6.2.8 treats the combination of bending and shear. This combination occurs frequently. The effect of shear force on the bending moment resistance should be taken into account. In many cases – especially when standard rolled sections are used – the effect of the shear force on the bending moment resistance is negligible. In fact, the influence of the shear force on the bending moment resistance may be neglected when the shear force is less than half of the design plastic shear resistance ($V_{Ed} \leq 0,5V_{pl,Rd}$), unless shear buckling governs the resistance of the cross-section (EN 1993-1-5, cl. 5). If the condition above is not satisfied – so $V_{Ed} > 0,5V_{pl,Rd}$ – the bending moment resistance should be determined with a reduced yield stress $f_{y,r}$ acting over the shear area according to:

$$f_{y,r} = \left(1 - \rho\right) f_y \quad \text{with} \quad \rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2 \qquad (7.37)$$

Rewriting this equation provides the relative reduced yield stress over the shear area:

$$\frac{f_{y,r}}{f_y} = 1 - \rho = 1 - \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2 \qquad (7.38)$$

Figure 7.8 shows the relationship between the relative reduced yield stress $f_{y,r}/f_y$, which acts over the shear area, and the utilization ratio for shear force $n = V_{Ed}/V_{pl,Rd}$.
If torsion is also present, the factor ρ should be determined from:

$$\rho = \left(\frac{2V_{Ed}}{V_{pl,T,Rd}} - 1\right)^2; \quad \text{but} \quad \rho = 0 \quad \text{for} \quad V_{Ed} \leq 0{,}5V_{pl,T,Rd} \qquad (7.39)$$

Where $V_{pl,T,Rd}$ is the reduced plastic design shear resistance allowing for the presence of torsion, see section 7.3.5 and equations (7.34) to (7.36).
For I-sections loaded in bending about the major axis (y-axis), a reduced design moment resistance ($M_{y,V,Rd}$) allowing for shear may be used as an alternative for calculating the reduced yield stress over the shear area:

$$M_{y,V,Rd} = \frac{\left(W_{pl,y} - \frac{\rho A_w^2}{4t_w}\right)f_y}{\gamma_{M0}} \leq M_{y,c,Rd} \qquad (7.40)$$

EN 1993-1-1, cl. 6.2.8 does not explicitly state how this equation should be used, it is however clear that the design bending moment should be limited to the reduced design moment resistance allowing for shear:

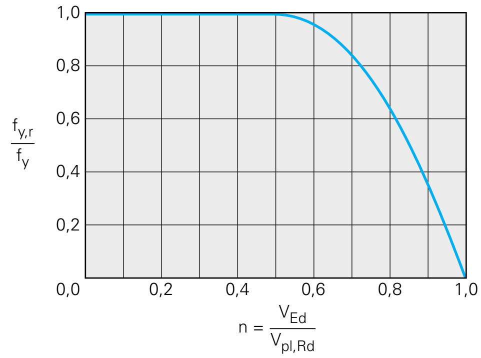
$$\frac{M_{Ed}}{M_{y,V,Rd}} \leq 1{,}0 \qquad (7.41)$$

Equation (7.40), for the reduced design moment resistance $M_{y,V,Rd}$ allowing for shear, can be derived as follows. The plastic section modulus in this equation is reduced, considering only that part of the plastic section modulus associated with the web. For a rectangular web, the plastic section modulus $W_{pl,web}$ is:

$$W_{pl,web} = \frac{1}{4}t_w h_w^2 = \frac{A_w^2}{4t_w} \qquad (7.42)$$

The bending moment resistance of a class 1 I-section with respect to the major axis is:

$$M_{pl,y,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} \qquad (7.43)$$



7.8 Relative reduced yield stress as a function of the utilization ratio n for shear force.